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Return of the EMC Effect

Gerald A. Miller and Jason R. Smith
Department of Physics
University of Washington
Seattle, WA 98195-1560

Abstract

The relationship between the properties of nuclear matter and structure functions measured in lepton-nucleus deep inelastic scattering is investigated using light front dynamics. We find that relativistic mean field models such as the Walecka, Zimanyi-Moszkowski (and point-coupling versions of the same) and Rusnak-Furnstahl models contain essentially no binding effect, in accord with an earlier calculation by Birse. These models are found to obey the Hugenholtz-van Hove theorem, which is applicable if nucleons are the only degrees of freedom. Any model in which the entire Fock space wave function can be represented in terms of free nucleons must obey this theorem, which implies that all of the plus momentum is carried by nucleons, and therefore that there will be essentially no binding effect. The explicit presence of nuclear mesons allows one to obtain a modified form of the Hugenholtz-van Hove theorem, which is equivalent to the often-used momentum sum rule. These results argue in favor of a conclusion that the depletion of the deep inelastic structure function observed in the valence quark regime is due to some interesting effect involving dynamics beyond the conventional nucleon-meson treatment of nuclear physics.

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I. INTRODUCTION

The European Muon Collaboration (EMC) effect in which the structure function of a nucleus, measured in deep inelastic scattering at values of Bjorken $x \geq 0.4$ corresponding to the valence quark regime, was found to be reduced compared with that of a free nucleon was discovered almost twenty years ago [1]. Despite much experimental and theoretical progress [2,3], no unique and universally accepted explanation of the depletion has emerged. The immediate parton model interpretation that the nucleon bound in a nucleus carries less momentum than in free space seems uncontested, but determining the underlying origin remains an elusive goal.

One popular explanation is that conventional nuclear binding effects are responsible. The conventional lore is that the nuclear structure function $F_{2A}(x)$ (which gives the momentum distribution of a quark in a nucleus as a function of the fractional momentum carried) can be obtained from the light front distribution function $f(y)$ (which gives the probability that a nucleon carries a fractional momentum y) and the nucleon structure function of a free nucleon F_{2N} using the relation [4]:

$$\frac{F_{2A}^{\rm lore}(x)}{A} = \int dy f(y) F_{2N}(x/y). \tag{1.1}$$

This formula has a simple interpretation as an expression which gives a probability as a sum of products of probabilities. The variable x is the Bjorken variable $x = Q^2/2M\nu$, and y is the A times the fraction of the nuclear plus-momentum carried by the nucleon, The plus component of a four vector is the sum of the time and third spatial component, so if k^μ is the momentum of a nucleon and P^μ is the momentum of the target nucleus $y = (k^0 + k^3)A/P^+ = (k^0 + k^3)A/M_A = (k^0 + k^3)/\overline{M}$, in which the nucleus is taken to be at rest with $P^+ = M_A$. One can easily use conventional nuclear physics to obtain the probability that a nucleon carries a three momentum $\mathbf{k}$, but, if one uses only naive considerations, one faces a puzzle when deciding how to choose the value of k^0. Should one use the average separation energy, or the average nucleon mass $\overline{M}$, or possibly the effective mass in the chosen many-body theory?

The essence of the binding explanation is that k^0 is given by the free nucleon mass M minus the average separation energy ϵ. Then $f(y)$ is narrowly peaked at $y = 1 - \epsilon/M$ (ϵ (with $\sim$ 70 MeV for infinite nuclear matter [5]). In this case, the structure function of a bound nucleon is approximately obtained by replacing $F_{2N}(x)$ by $F_{2N}(x/(1 - \epsilon/M))$. The increase in the argument leads to a significant reduction in the value of the nuclear structure function The theoretical understanding of the binding effect (as of 1996) is reviewed nicely in the book [6], which summarizes the various treatments as "not completely satisfactory". This kind of explanation seems very natural because nuclear binding is known to occur, so such an effect must be understood thoroughly before hoping to extract information about a possible host of more interesting exotic effects. In any case, one needs to supply a derivation to avoid the need to arbitrarily choose a prescription for k^0.

This need drove one of us on to the light front [7,8]. That is, to attempt to use light front dynamics to derive the nuclear wave function. The reason for this is that, in the parton model x is the ratio of the plus component of the momentum of the struck quark to that of the target, and it is the plus component of the momentum which was observed to be depleted by the EMC. In the view of Ref. [9], using light front dynamics is the most effective way to assess the influence of binding effects. However, one must pay the price of computing nuclear wave functions using these dynamics.

The first attempts [7,8] in this direction employed the popular and successful Walecka model [10] which has many effective descendants [11–13]. The salient result was that vector mesons carried 35% of the nuclear plus momentum and nucleons only 65% ($P_N^+/P^+ = 0.65$), far smaller than the value $(1 - \epsilon/M) \sim 0.95$ needed to reproduce the observations for the iron nucleus. However, the connection between the nucleon momentum distribution computed using light front dynamics and that used in computing the deep inelastic structure function was not made. Recently, the authors of Ref. [14] have claimed that quark distribution functions are not parton probabilities. Their message to us is that, in any situation, one needs to derive the connection between the constituent distribution function and the observed data. That work stimulated us to undertake the present investigation in which we derive the connection between the nucleon momentum distribution and the structure function measured in deep inelastic scattering.

First we outline our procedure. We start in Section II by considering relativistic models of infinite nuclear matter computed using the mean field approximation. We derive and apply the nucleon distribution function $f_N(y)$ appropriate for use in computing deep inelastic scattering structure functions. The function $f_N(y)$ is shown to be the one which maintains the covariance of the formalism, and in which the nucleons carry the entire plus-momentum, P^+ of the nucleus [15]. This result is obtained independently of the specific relativistic mean field theory used, so no such theory contains the binding effect discussed above. The only binding effect arises from the average binding energy of the nucleus (16 MeV for infinite nuclear matter), and is far too small to explain the observed depletion of the structure function. This is in accord with an earlier similar finding by Birse [16]. The generality of this result encourages us to seek a broader context. This is found in the Hugenholtz-van Hove theorem [17] which states that the binding energy of the level at the Fermi surface is equal to the average binding energy, or the energy of the level at the Fermi surface E_F is equal to the nuclear mass divided by A:

$$E_F = M_A/A \equiv \overline{M}. \tag{1.2}$$

This theorem is the consequence of using the condition that the total pressure of the nucleus vanishes at equilibrium, and the assumption that nucleons are the only degrees of freedom contributing to the nuclear energy. Thus this theorem is a signal that $P^+ = P_N^+$ or that nucleons account for the entire plus momentum of the nucleus. This generally is understood to imply that there will be no EMC binding effect [3], thus any model which obeys Eq. (1.2) can be expected not to have one.

The next step is to recall in Section III how light front dynamics is applied to computing the properties of infinite nuclear matter using the Walecka model (as a specific example) in mean field approximation (MFA). The purpose is to illustrate the general formalism needed to go beyond the mean field approximation, provide an explicit example of the general results presented in Section II, study the nuclear structure origins of the nuclear momentum content, and show explicitly that the Hugenholtz-van Hove theorem is satisfied.

In Section IV we introduce four other model Lagrangians in which the values of the effective mass and vector meson field vary widely. Again our specific calculations are limited to the MFA. However, in Section V, the application of the Hugenholtz-van Hove theorem [17] allows us to make some general statements about models which include nucleon-nucleon correlations. In particular, we use this theorem to explain why no binding effect is contained in any model, such as that of Ref. [18], in which nucleons are responsible for the entire plus momentum of the nucleus. This is in accord with early observations of Ref. [3], but now there is an additional ability to compute all of the relevant nuclear properties using light front dynamics. We also use our findings assess existing treatments of the binding effect. Section VI is a summary of our results and their implications. In Appendix A we use light front dynamics to compute nuclear properties of the four models of Section IV.

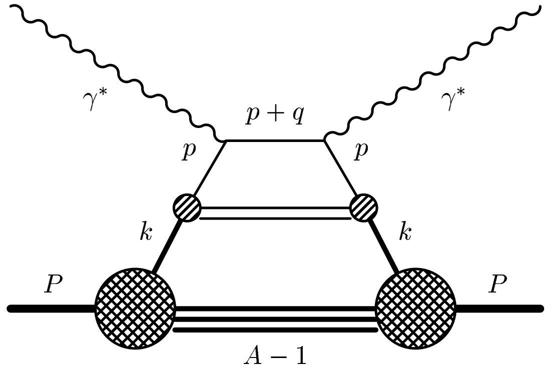

FIG. 1. Diagram for computing the nuclear structure function. A nucleus of momentum P emits a nucleon of momentum k, which emits a quark of momentum p, which absorbs the virtual photon of momentum q.

II. DEEP INELASTIC SCATTERING FROM NUCLEI

We are testing the hypothesis that conventional nuclear dynamics can explain the EMC effect. This means that we need to include possible binding energy and Fermi motion effects, but not dynamics related to true modifications of the nucleon structure or off-shell effects caused by the nuclear medium. The key assumption is that the system formed by the absorption of the photon is not a bound nucleon and therefore does not have the same interaction. The relevant lifetime of the struck system is $\frac{1}{xM} \leq 0.5$ fm (for $x \geq 0.5$) which corresponds to a very short nuclear time, too short for interactions. In this case the use of a manifestly covariant formulation to derive the expression for the structure function leads to a convolution formula. If one uses the free nucleon structure function (neglecting off-shell effects) and F_{2N} for free nucleons one finds [19]

$$\frac{F_{2A}(x_A)}{A} = \int_{x_A}^{\infty} dy f_N(y) F_{2N}(x_A/y), \tag{2.1}$$

$$f_N(y) = \int \frac{d^4k}{(2\pi)^4} \delta(y - \frac{k^0 + k^3}{\overline{M}}) Tr \left[\frac{\gamma^+}{2P^+A} \chi(k,P) \right], \tag{2.2}$$

where P is the total four momentum of the nucleus, and

$$x_A \equiv Q^2 A / 2P \cdot q = xAM/M_A = xM/\overline{M} \tag{2.3}$$

with M as the free nucleon mass and $\overline{M} \equiv M_A/A$. The function $\chi(k,P)$ is (proportional to) the connected part of the nuclear expectation value of the nucleon Green's function [20], and the trace is over the Dirac and isospin indices. That $f_N(y)$ is a Lorentz scalar is manifest from the structure of Eq. (2.2). We note the appearance of $\overline{M}$ instead of the free value of the nucleon mass M. This arises here from the definition (2.3) and the feature that, in the Bjorken limit, the nuclear structure function depends on the ratio $p^+/P^+ = (p^+/k^+)(k^+/P^+)$, where p^μ is the quark momentum, with $P^+ = M_A = A\overline{M}$. The basis of the formula (2.2) is that both the quark and nucleon distributions are directly related to manifestly covariant Green's functions [19,21]. This is a standard result using nothing more than the stated assumptions and the Feynman diagram in Fig. 1.

The manifestly covariant form of the single nucleon Green's function has been known for a long time [22], and its use (in the nucleus rest frame) leads to the result

$$\chi(k,P) = -i2P^+\Omega\,(\gamma\cdot(k-g_vV)+M^*) \times \left[\frac{1}{(k-V)^2-M^{*2}+i\epsilon}+\frac{i\pi}{E^*(k)}\delta(k^0-E^*(k)-g_vV^0))\theta(k_F-|\mathbf{k}|)\right], \tag{2.4}$$

where

$$E^*(k) \equiv \sqrt{M^{*2}+\mathbf{k}^2}. \tag{2.5}$$

The general form of the Green's function depends on a vector potential $V=(V^0,\mathbf{0})$ for a nucleus at rest, and the effective mass M^* which includes the effects of interactions on the nucleon mass. The values of V and M^* depend on the specific Lagrangian employed, but the form of the Green's function is general. Recall also that $V^-=V^+=V^0$ for the expectation values of vector meson fields in the nucleus rest frame.

The result (2.4) was first obtained using the conventional equal time approach, but the very same can also be obtained from the light front formalism. In that case it is necessary to include the effects of the instantaneous part of the nucleon Green's function and those of the instantaneous meson exchange.

The next step is to insert the connected part (second term) of (2.4) into Eq. (2.2) for $f_N(y)$. This gives, after taking the trace and using the delta function to integrate over k^0, the result

$$f_N(y) = \frac{4}{(2\pi)^3\rho_B}\int d^2k_\perp\,dk^3\frac{E^*(k)+k^3}{E^*(k)}\delta(y-\frac{E^*(k)+g_vV^++k^3}{\overline{M}})\theta(k_F-|\mathbf{k}|). \tag{2.6}$$

The integration is simplified by using the transformation

$$k^+ \equiv E^*(k)+k^3, \tag{2.7}$$

which makes a connection with light front variables [23]. It is an exercise in geometry to show that the Fermi volume can be re-expressed in terms of k^+ using

$$k_\perp^2+(k^+-E_F^*)^2 \le k_F^2, \quad E_F^* \equiv \sqrt{k_F^2+M^{*2}}, \tag{2.8}$$

so that Eq. (2.6) becomes

$$f_N(y) = \frac{4}{(2\pi)^3\rho_B}\int d^2k_\perp\int dk^+\theta\left(k_F^2-k_\perp^2-(k^+-E_F^*)^2\right)\delta(y-\frac{k^++g_vV^+}{\overline{M}}). \tag{2.9}$$

The use of the definition of the energy of a nucleon at the Fermi surface,

$$E_F = E_F^*+g_vV^+ = E_F^*+g_vV^0, \tag{2.10}$$

allows one to achieve a simple expression for $f_N(y)$:

$$f_N(y) = \frac{3}{4}\frac{\overline{M}^3}{k_F^3}\theta((E_F+k_F)/\overline{M}-y)\theta(y-(E_F-k_F)/\overline{M}))\left[\frac{k_F^2}{\overline{M}^2}-(\frac{E_F}{\overline{M}}-y)^2\right]. \tag{2.11}$$

The result Eq. (2.11) can be further simplified by using the Hugenholtz-van Hove theorem displayed in Eq. (1.2). Section III contains an explicit demonstration of Eq. (1.2) for the Walecka model and the Appendix contains a similar demonstration for the other relativistic models evaluated using the mean field approximation. Using Eq. (1.2) in Eq. (2.11) therefore leads to the general result

$$f_N(y) = \frac{3}{4}\frac{\overline{M}^3}{k_F^3}\theta(1 + k_F/\overline{M} - y)\theta(y - (1 - k_F/\overline{M}))\left[\frac{k_F^2}{\overline{M}^2} - (1-y)^2\right], \tag{2.12}$$

correct for any relativistic mean field theory of infinite nuclear matter. Different theories with the same binding energy and Fermi momentum may have very different scalar and vector potentials, but must have the same $f_N(y)$.

A result very similar to Eq. (2.12) was previously obtained by Birse [16]. The difference between his formula and ours is the appearance of $\overline{M}$ in the function $f_N(y)$, whereas he uses M. This difference is a small effect numerically, and therefore our conclusions will be the same as his.

The baryon sum rule and momentum sum rules are derived by taking the first two moments of $f_N(y)$. This gives:

$$\int dy f_N(y) = 1 \tag{2.13}$$

$$\int dy\; y\; f_N(y) = 1. \tag{2.14}$$

The latter equation is remarkable; it states that in deep inelastic scattering the nucleons act as if they carry all of the P^+ of the nucleus even though the mesonic fields are very prominent.

This is clearer if we re-interpret these sum rules in terms of a probability $f_N(k^+)$ that a nucleon has a plus momentum $k^+ \equiv y\overline{M}$, with $f_N(k^+) \equiv A f_N(y\overline{M})/\overline{M}$, so that

$$\int dk^+\; f_N(k^+) = A, \tag{2.15}$$

$$\int dk^+\; k^+\; f_N(k^+) = A\overline{M} = M_A \tag{2.16}$$

The momentum sum rule (2.16) shows the total plus momentum carried by the nucleons (as seen in deep inelastic scattering) is also the total momentum carried by the nucleus.

The main result of this is that the nuclear structure function is given by Eq. (2.2) with the function $f_N(y)$ obtained in Eq. (2.12). This tells us that, despite the fact that there is considerable binding energy, there is no EMC binding effect. Indeed, F_{2A} depends on the Fermi momentum but does not depend on the effective mass M^*.

The quantity measured in deep inelastic scattering is the ratio defined by

$$R(x) = \frac{F_{2A}(x_A)}{AF_{2N}(x)}. \tag{2.17}$$

A numerical study of this expression using, five different relativistic models is presented below. First, we emphasize the qualitative features. Since the width of f_N is given by

TABLE I. Summary of the Models-Taking the sum of $g_v V^+/\overline{M}$ and $E_F^*/\overline{M}$ shows that each model satisfies the Hugenholtz-van Hove theorem, Eq. (1.2).

Model	$g_v V^+$	$g_v V^+/\overline{M}$	$E_F^*/\overline{M}$	M^*/M	k_F (fm^{-1})	P_N^+/P_A^+
W	$g_v^2\rho_B/m_v^2$	0.355	0.645	0.56	1.42	0.65
NVW	$2G\rho_B$	0.355	0.645	0.56	1.42	1
ZM	$g_v^2\rho_B/m_v^2$	0.079	0.921	0.85	1.42	0.92
NVZM	$2G\rho_B$	0.079	0.921	0.85	1.42	1
RF	$2G_{RF}\rho_B$	0.194	0.806	0.73	1.31	1

the small quantity $k_F/\overline{M}$ it is a very narrow function. In this case, one may evaluate the integrand of Eq. (2.2) by expanding $F_{2N}(x/y)$ in a Taylor series about $y = 1$ [24] to find that

$$R(x) = \frac{F_{2N}(x_A)}{F_{2N}(x)} + \frac{k_F^2}{10\overline{M}^2 F_{2N}(x)} \left(2x_A F_{2N}'(x_A) + x_A^2 F_{2N}''(x_A)\right), \tag{2.18}$$

which shows that the only effect of the binding energy occurs in the small difference between x_A and x which depends only on the small average binding energy. Note that a term proportional to $F_{2N}'(x)$ (but not proportional to the small parameter $\frac{k_F^2}{\overline{M}^2}$)) vanishes because one is expanding about $y = 1$ and using the baryon and momentum sum rules Eqs. (2.15,2.16). We may further approximate $R(x)$ by expanding the first term about the value $x_A = x$ ($x_A = 1.02\,x$ for nuclear matter, and $x_A \leq 1.01x$ for finite nuclei). Thus

$$R(x) = 1 + \frac{\langle\epsilon\rangle}{\overline{M}} \frac{F_{2N}'(x)}{F_{2N}(x)} + \frac{k_F^2}{10\overline{M}^2 F_{2N}(x)} \left(2x_A F_{2N}'(x_A) + x_A^2 F_{2N}''(x_A)\right), \tag{2.19}$$

where $\langle\epsilon\rangle$ is the binding energy per nucleon (16 MeV for infinite nuclear matter and ≤ 8 MeV for finite nuclei). This shows that, as long as the Hugenholtz-van Hove theorem is applicable, the only binding effect is due to the binding energy per nucleon. One is not allowed to use the separation energy which is much larger. The use of Eq. (2.19) cannot lead to a large enough depletion of $R(x)$ [16] to resemble the extrapolated data for nuclear matter [25].

The relevant parameters of these models are displayed in Table I. The qualitative features discussed above are prominent in the numerical calculations displayed in Fig. 2 in which the ratio $R(x)$ of Eq. (2.17) is presented for five different relativistic models. The relevant parameters of these models are displayed in Table I. We note that four of the models have identical results, with only the Rusnak-Furnstahl model (with its chosen different value of k_F) differing only very slightly. These results are obtained using a simple early parameterization [26] of the free nucleon structure function

$$F_{2N}(x) = 0.58\sqrt{x}(1-x)^{2.8} + 0.33\sqrt{x}(1-x)^{3.8} + 0.49(1-x)^8, \tag{2.20}$$

but the essential features of the curves are independent of the free nucleon structure function.

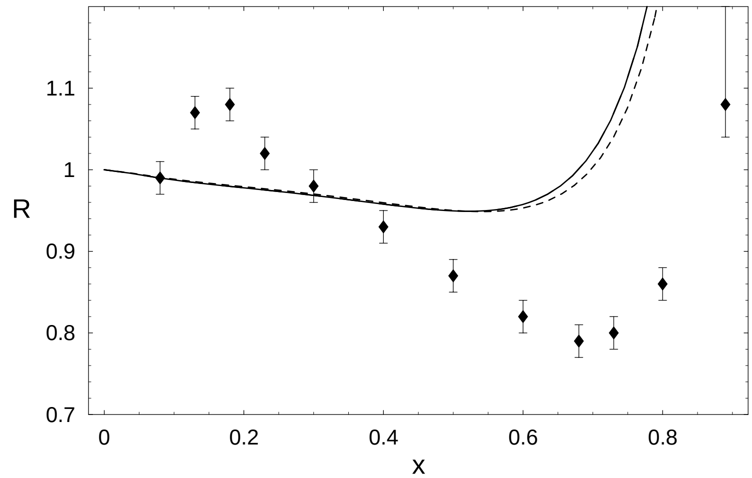

FIG. 2. $R(x)$ vs. x for W, NVW, ZM, NVZM (solid line) and RF (dashed line). The data shown here are from the extrapolation of Ref. [25]

III. THE CANONICAL FORMALISM–WALECKA MODEL

We illustrate the canonical light front formalism using the Walecka model as our first example. Some of this has been published previously [8], but our purposes here are to set up and illustrate the formalism necessary to go beyond the mean field approximation, provide an explicit example of the general results presented in Section II, study the nuclear structure origins of the nuclear momentum content, and show explicitly that the Hugenholtz-van Hove theorem is satisfied.

The Walecka model employs a Lagrangian containing fields for nucleons (ψ'), scalar mesons ϕ and vector mesons V^μ:

$$\mathcal{L}_W = \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi - m_s^2\phi^2) - \frac{1}{4}V^{\mu\nu}V_{\mu\nu} + \frac{1}{2}m_v^2 V^\mu V_\mu + \overline{\psi}'\left(\gamma^\mu(i\partial_\mu - g_v V_\mu) - M - g_s\phi\right)\psi', \quad (3.1)$$

with the field equations:

$$(\partial_\mu\partial^\mu + m_s^2)\phi = -g_s\overline{\psi'}\psi' \quad (3.2)$$

$$\partial_\mu V^{\mu\nu} + m_v^2 V^\nu = g_v\overline{\psi}'\gamma^\nu\psi' \quad (3.3)$$

$$\gamma^\mu(i\partial_\mu - V_\mu)\psi' = (M + g_s\phi)\psi'. \quad (3.4)$$

The symmetric canonical energy-momentum tensor is given by [27–29]

$$T^{\mu\nu} = -g^{\mu\nu}\mathcal{L} + V^{\alpha\mu}V^{\beta\nu}g_{\beta\alpha} + \partial^\mu\phi\partial^\nu\phi + \frac{1}{2}\overline{\psi'}(\gamma^\mu(i\partial^\nu - g_v V^\nu) + \gamma^\nu(i\partial^\mu - g_v V^\mu))\psi'. \quad (3.5)$$

A. Mean Field Approximation for Infinite Nuclear Matter

We follow the MFA [10] in assuming that the sources are sufficiently strong so that the resulting large numbers of mesons can be treated in a classical manner in which source operators are replaced by their expectation values in the nuclear ground state. Furthermore, for a system of infinite volume, all positions and directions (in the nuclear rest frame) are equivalent. In this case ϕ and V^0 are constants and $\mathbf{V} = \mathbf{0}$. The approximate mesonic equations of motion become

$$\phi = -\frac{g_s}{m_s^2}\langle \overline{\psi'}\psi' \rangle = -\frac{g_s}{m_s^2}\rho_s \tag{3.6}$$

$$V^0 = V^{\pm} = \frac{g_v}{m_v^2}\langle \psi'^{\dagger}\psi' \rangle = \frac{g_v}{m_v^2}\rho_B, \tag{3.7}$$

in which the brackets are used as an abbreviation for taking the ground state matrix element, and

$$\rho_B = 2k_F^3/3\pi^2. \tag{3.8}$$

The nucleon, though described in terms of four-component spinors, consists of only two independent fields. The independent and dependent degrees of freedom are defined by the projection operators: $\Lambda_{\pm} \equiv \frac{1}{2}\gamma^0\gamma^{\pm}$, $\psi'_{\pm} \equiv \Lambda_{\pm}\psi'$, with ψ'_+ chosen as the independent field. One more step is necessary because the resulting equation for ψ'_- depends on V^+ in a complicated manner. It is traditional in light front dynamics of mass-less vector bosons to remove the effects of the term V^+, by working in a gauge with $V^+ = 0$. Here we use the Soper-Yan transformation [27,29]:

$$\psi' \equiv e^{-ig_v\Lambda}\psi, \qquad \partial^+\Lambda = V^+, \tag{3.9}$$

which allows a simple equation for ψ'_- in terms of ψ'_+ to proceed in a satisfactory manner, but which also causes the loss of manifest covariance. With this transformation the final version of the nucleon field equation becomes

$$\begin{aligned} (i\partial^- - g_vV^-)\psi_+ &= (\boldsymbol{\alpha}_\perp \cdot \mathbf{p}_\perp + \beta(M + g_s\phi))\psi_- \\ i\partial^+\psi_- &= (\boldsymbol{\alpha}_\perp \cdot \mathbf{p}_\perp + \beta(M + g_s\phi))\psi_+ \end{aligned} \tag{3.10}$$

within the mean field approximation (in which $\partial^-\Lambda = 0$) for infinite nuclear matter. The nucleon mode functions are plane waves so $\psi \sim e^{ik\cdot x}$ and

$$(i\partial^- - g_vV^-)\psi_+ = \frac{k_\perp^2 + M^{*2}}{k^+}\psi_+, \tag{3.11}$$

where $M^* \equiv M + g_s\phi$.

The relevant components of the energy-momentum tensor in the mean field approximation MFA are obtained by using the constant meson fields of Eq. (3.6) and (3.7) in Eq. (3.5) to obtain

$$T^{++}_{MFA} = m_v^2V_0^2 + 2\psi_+^{\dagger}i\partial^+\psi_+ \tag{3.12}$$

$$T^{+-}_{MFA} = m_s^2\phi^2 + 2\psi_+^{\dagger}(i\partial^- - g_vV^-)\psi_+ \tag{3.13}$$

and so P^+ and P^- are given by

$$P^{\pm} = \langle T^{+\pm}_{MFA} \rangle \Omega, \tag{3.14}$$

where Ω is the volume of the system, taken as infinite at the end of the calculation ($A, \Omega \to \infty$ with A/Ω finite). The evaluation of these expectation values yields

$$\frac{P^-}{\Omega} = m_s^2 \phi^2 + \frac{4}{(2\pi)^3} \int_F d^2k_\perp dk^+ \frac{k_\perp^2 + (M^*)^2}{k^+} \tag{3.15}$$

$$\frac{P^+}{\Omega} = m_v^2 V_0^2 + \frac{4}{(2\pi)^3} \int_F d^2k_\perp dk^+ k^+. \tag{3.16}$$

It is necessary to define the Fermi sea within the present context. Although we do not have manifest rotational invariance here, this invariance is restored in the results if we define the component k^3 implicitly through Eq. (2.7). Then

$$\int_F d^3k \cdots \equiv \int d^3k \theta(k_F - k) \cdots, \tag{3.17}$$

and geometry leads to

$$\int_F d^2k_\perp dk^+ \cdots \equiv \int d^2k_\perp dk^+ \theta\left(k_F^2 - k_\perp^2 - (k^+ - E_F^*)^2\right) \cdots. \tag{3.18}$$

Using Eqs. (3.15-3.17) leads to the results that the value of the energy of the system in the rest-frame,

$$E_A \equiv \frac{1}{2}(P^+ + P^-), \tag{3.19}$$

is the same as in the usual treatment of the Walecka model, as shown below. The only remaining task is to determine the Fermi momentum, k_F. This is done by using the minimization

$$\left(\frac{\partial (E_A/A)}{\partial k_F}\right)_\Omega = 0, \tag{3.20}$$

$$E_A(k_F) = M_A. \tag{3.21}$$

Carrying out the differentiation leads to an equation which is equivalent to setting $P^+ = P^-$, which must occur for a system in its rest frame with $P^3 = 0$. Since rotational invariance is maintained in the solution $P^{1,2,3} = 0$, and therefore the pressure $P = 1/3 \sum_{i=1,3} P^i$ vanishes. Thus the equation $P^+ = P^-$ is also the light front equivalent of setting the pressure P to 0. Note also that one may explicitly carry out the differentiation to find that

$$E_A/A = M_A/A = E_F^* + g_v V^0 = E_F \tag{3.22}$$

which is the Hugenholtz-van Hove theorem [17].

The above paragraph is serves as an outline of the derivation of the Hugenholtz-van Hove theorem, but we also provide an explicit proof. First, use the transformation (2.7) to obtain the results

$$\frac{P^-}{\Omega} = m_s^2\phi^2 + \frac{4}{(2\pi)^3}\int_F d^3k(E^*(k) - \frac{1}{3}\boldsymbol{k}\cdot\boldsymbol{k}) \tag{3.23}$$

$$\frac{P^+}{\Omega} = m_v^2V_0^2 + \frac{4}{(2\pi)^3}\int_F d^3k(E^*(k) + \frac{1}{3E^*(k)}\boldsymbol{k}\cdot\boldsymbol{k}) \tag{3.24}$$

$$\frac{E_A}{\Omega} = \frac{1}{2}m_s^2\phi^2 + \frac{1}{2}m_v^2V_0^2 + \frac{4}{(2\pi)^3}\int_F d^3kE^*(k) \tag{3.25}$$

Next carry out the differentiation in Eq. (3.20), using $A = \rho_B\Omega$ to obtain

$$\frac{\partial E_A}{\partial k_F} = 3\frac{E_A}{k_F}. \tag{3.26}$$

The term $\frac{\partial E_A}{\partial k_F}$ is obtained by first eliminating all derivatives with respect to ϕ using the feature that setting $\frac{\partial E_A}{\partial \phi}$ to zero reproduces the field equation for ϕ. Then one uses the field equation for the vector meson (3.7). The result is

$$\frac{4}{(2\pi)^3}\frac{4\pi}{3}k_f^3E_F^* = \frac{m_s^2}{2}\phi^2 - \frac{m_v^2}{2}V_0^2 + \frac{4}{(2\pi)^3}\int_F d^3k\ E^*(k), \tag{3.27}$$

This is a transcendental equation which determines k_F, so that the calculation of E_A is complete. With the self-consistent Eqs. (3.6,3.7,3.25) one obtains an average binding energy of 15.75 MeV with a Fermi momentum of $k_F = 1.42$ fm^{-1} using the parameters: $\frac{g_v^2}{m_v^2}M^2 = 195.9, \frac{g_s^2}{m_s^2}M^2 = 267.1$, which corresponds to $g_vV^- = 323$ MeV, and $M^*/M = 0.56$. These parameters are the same as in the original Walecka model.

The relation $P^+ = P^-$ (which must hold for a system in its rest frame) also emerges as a result of this minimization. To see this, rewrite the left hand side of Eq. (3.27) as

$$\frac{4}{(2\pi)^3}\frac{4\pi}{3}k_f^3E_F^* = \frac{4}{(2\pi)^3}\int_F d^3k\left(E^*(k) + \frac{\boldsymbol{k}\cdot\boldsymbol{k}}{3E^*(k)}\right). \tag{3.28}$$

Using this in Eq. (3.27) leads to

$$\frac{m_s^2}{2}\phi^2 - \frac{m_v^2}{2}V_0^2 = \frac{4}{(2\pi)^3}\int_F d^3k\frac{\boldsymbol{k}\cdot\boldsymbol{k}}{3E^*(k)}, \tag{3.29}$$

which is what one obtains by setting $P^+ = P^-$ using Eqs. (3.23) and (3.24).

We now have the tools at hand to prove the Hugenholtz-van Hove theorem. Simply use Eq. (3.27) to remove the integral appearing in Eq. (3.25) and obtain

$$\frac{E_A}{\Omega} = m_v^2V_0^2 + \rho_BE_F^*. \tag{3.30}$$

Then the use of the field equation (3.7) yields

$$\frac{E_A}{\rho_B\Omega} = \frac{E_A}{A} = g_vV^0 + E_F^* = E_F, \tag{3.31}$$

which is the desired result. This is a remarkable result. The original version of the theorem was proved using only the assumption that nucleons are the only degrees of freedom. Here, the mesons are important, yet the theorem still holds [30].

B. Nuclear Plus-Momentum Content

Now we relate the role of the plus component of the momentum seen here with that of Section II.The nucleonic contribution to the nuclear plus momentum from Eq. (3.16) is

$$\frac{P_N^+}{A} = \frac{4}{\rho_B(2\pi)^3}\int_F d^2k_\perp dk^+ k^+, \tag{3.32}$$

which is also obtainable directly from taking the nuclear expectation value of nucleon plus momentum operator.

The large vector potential and small value of M^* are associated with the startling result that only 65% of the plus momentum of the nucleus is carried by nucleons, and that 35% is carried by vector mesons. It was previously argued [7,8] that this would produce a disastrously large decrease in the nuclear deep inelastic structure function. As shown above, that does not occur, because the function $f_N(y)$ peaks at $y = 1$.

We therefore need to understand how it is that the nucleons can carry 65% of the momentum here all of the momentum as stated in Section II. To do this, use Eq. (3.32) to define a probability $f(k^+)$ that a nucleon carries a plus momentum k^+:

$$P_N^+/A = \int dk^+ \ k^+ f(k^+), \tag{3.33}$$

with

$$f(k^+) = \frac{4}{\rho_B(2\pi)^3}\int_F d^2k_\perp = \frac{4}{\rho_B(2\pi)^3}\int_F d^2k_\perp dp^+ \delta(k^+ - p^+). \tag{3.34}$$

It is useful to again obtain a dimensionless distribution function $f(y)$ by replacing k^+ by the dimensionless variable y using $y \equiv \frac{k^+}{\overline{M}}$, $f(y) \equiv \overline{M} f(k^+)$. Then one finds

$$f(y) = \frac{3}{4}\frac{\overline{M}^3}{k_F^3}\theta(y^+ - y)\theta(y - y^-)\left[\frac{k_F^2}{\overline{M}^2} - (\frac{E_F^*}{\overline{M}} - y)^2\right], \tag{3.35}$$

where $y^\pm \equiv \frac{E_F^* \pm k_F}{\overline{M}}$. This function peaks at $y = E_F^*/\overline{M} = 0.65$ for the Walecka model, and the average value of y is also 0.65. Its use in Eq. (2.1) would indeed lead to a disaster. Indeed, our previous work assumed that Eq. (1.1) was appropriate. In that case, the computed ratio $\frac{F_{2A}(x)}{A}$ was dramatically smaller than F_{2N}. This disastrous result can be understood from the following logic. A reasonable first approximation to the integral Eq. (1.1) can be obtained by using $f(y) \approx \delta(y - \frac{E_F^*}{\overline{M}})$ which satisfies the baryon sum rule and corresponds to an average value of $y = 0.65$. Then $\frac{F_{2A}(x)}{A}$ vanishes for $x > 0.65$ and the ratio to the free structure function goes to zero in huge contradiction with experiment, which shows depletions no larger than 20% for the heaviest nuclear targets. This result is illustrated with the numerical calculation shown in Fig. 3.

However, the correct quantity to use in deep inelastic scattering is $f_N(y)$, which emerges from a manifestly covariant treatment. To see the connection between $f(y)$ and $f_N(y)$, it is first helpful to compare directly Eqs. (2.12) and (3.35). This comparison yields:

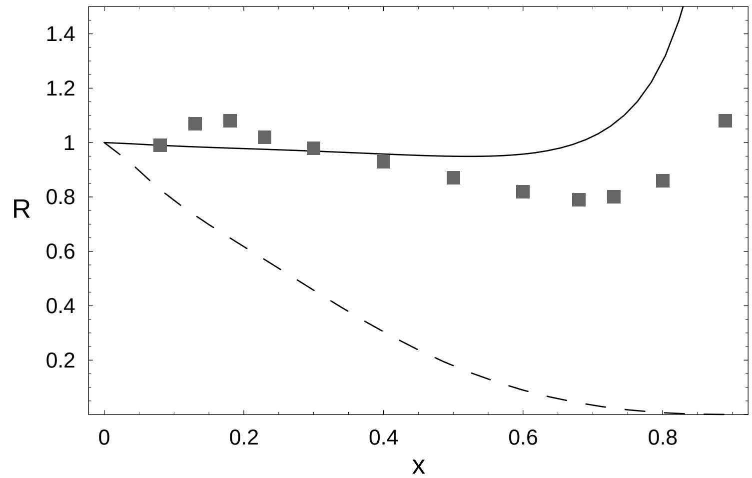

FIG. 3. $R(x)$ vs. x for the Walecka model using $f_N(y)$ (solid line) and $f(y)$ in Eq. (2.1) (dashed line). The data shown here are from the extrapolation of Ref. [25]

$$f(y) = f_N(y + g_v V^+/\overline{M}), \tag{3.36}$$

using the Hugenholtz-van Hove theorem Eq. (1.2). The correct nucleon distribution function is one that is shifted by the vector potential. This simple relation suggests that there is a simple interpretation of the difference between $f_N(y)$ and $f(y)$ in terms of a phase difference. Indeed, the difference arises from the Soper-Yan transformation (3.9) which relates the fields ψ and ψ'. Thus we have two forms of the plus momentum density, T^{++}_{MFA}:

$$T^{++}_{MFA} = m_v^2 V_0^2 + 2\psi_+^\dagger i\partial^+ \psi_+ = 2\psi'^\dagger{}_+ \left(i\partial^+ - g_v V^+\right)\psi'_+. \tag{3.37}$$

In the second form, a nucleon operator carries all of the plus momentum, the correct single nucleon plus momentum operator is the canonical conjugate momentum which is shifted by the term $g_v V^+$. The second form is the appropriate one as it is related to the original covariant Lagrangian. The term $y f_N(y)$ is obtained from the expectation value of the nucleon plus-momentum operator $\psi_+'^\dagger(i\partial^+ - g_v V^+)\psi'_+$, while $yf(y)$ is obtained from $\psi_+^\dagger i\partial^+\psi_+$ with $f_N(y)$ as the distribution function which emerges from a covariant treatment.

IV. FOUR MORE MODELS IN MEAN FIELD APPROXIMATION

The Walecka model, evaluated in MFA, was known to have some phenomenological troubles. The compressibility is too large, and the very small effective mass (shown here to be irrelevant for deep inelastic scattering) does enter into quasi-elastic scattering, in which it is a straightforward matter to show that the cross section is given by an integral over the distribution, $f(y)$, and not $f_N(y)$. The reason for this is that in the MFA the struck nucleon feels the same vector and scalar potentials as a bound nucleon. Hence it was of interest to improve the Lagrangian. This has been done in a variety of ways. We consider four other models here, mainly to show that the same function $f_N(y)$ emerges from each one and that each one satisfies the Hugenholtz-van Hove theorem. The validity of this theorem is a signal that the nucleons carry all of the plus-momentum, just as for the Walecka model, so that there can be no significant binding effect.

All of the models have essentially the same saturation properties, but each is distinguished by using a different mechanism to reduce the putative amount of momentum carried by the vector mesons. Here we simply define the models and summarize the results. The details of the solution are presented in the Appendix. The important parameters of each model are displayed in Table I.

The No Vector Walecka (NVW) model is defined by the elimination of the vector meson field in favor of a point coupling interaction of the form $Gj^\mu j_\mu$ with $j_\mu \equiv \overline{\psi'}\gamma_\mu\psi'$. In this case the nucleons carry all of the momentum. The Zimanyi-Moszkowski (ZM) model [31] is defined by using a rescaled derivative coupling interaction in which the scalar coupling in the Lagrangian is given by the term $-\overline{\psi}'M/(1-\frac{g_s\phi}{M})\psi'$. This model is known to involve a larger effective mass and smaller vector field than the Walecka model. The Appendix shows that, in this model, the nucleons carry about 92% of the total plus momentum. The No Vector Zimanyi-Moszkowski (NVZM) model is defined by starting with the Zimanyi-Moszkowski Lagrangian and then removing the vector mesons in favor of a current-current interaction as in the NVW model. Again nucleons carry all of the plus-momentum. The Rusnak-Furnstahl (RF) point coupling model contains no explicit meson fields, and the interactions included via a variety of non-linear couplings. The parameters are given in Ref. [12], and arguments for their naturalness in terms of effective field theory have also been presented [13]. The nucleons carry all of the plus-momentum and the numerical value of the effective mass is $M^* = 0.73M$, significantly higher than that of the Walecka model.

Each of the models summarized in Table I has essentially the same saturation properties even though the values of M^* and E_F^* display huge variations. Note that each model has a nucleon mode equation (3.11), (A9), (A16), (A29), and (A36), and these are summarized by the single unifying expression:

$$(i\partial^- - g_v V^+)\psi_+ = \frac{k_\perp^2 + (M^*)^2}{k^+}\psi_+. \tag{4.1}$$

The numerical values of $g_v V^+$ are listed along with other relevant information regarding the five models in Table I.

V. BEYOND THE MEAN FIELD APPROXIMATION

It is worthwhile to discuss the generality of the result that there is no significant binding effect. Consider any model, such that of Ref. [18] in which mesonic fields are not explicit components of the nuclear Fock state wave function. For example, one may eliminate the mesonic degrees of freedom in favor of two- and three- nucleon interactions without maintaining the mesonic presence in the nuclear Fock state wave function. Such models, correctly evaluated, obey the Hugenholtz-van Hove theorem, Eq. (1.2). The validity of this theorem is a signal that nucleons carry all of the plus momentum, so that the baryon and momentum sum rules Eqs. (2.13-2.16) are satisfied. This means that for equilibrium the following conditions hold

$$P^\pm = P_N^\pm \tag{5.1}$$
$$P^+ = P^-. \tag{5.2}$$

These two equations (the first is defined by the model, the second by stability) may be thought of as the light front version of the Hugenholtz-van Hove theorem. Therefore one may again apply the analysis of Refs. [3,24] and expand $f_N(y)$, appearing in the integral of Eq. (2.2) about its peak value of unity. It is generally sufficient to keep only three terms. Thus one finds

$$F_{2A}(x) = F_{2N}(x_A) + \gamma\left(2x_A F'_{2N}(x_A) + x_A^2 F''_{2N}(x_A)\right) \tag{5.3}$$

$$\gamma \equiv \int dy\ (y-1)^2 f_N(y). \tag{5.4}$$

The coefficient γ is larger than the term proportion to k_F^2 of Eq. (2.19) because the effects of correlations extend the width of the distribution $f_N(y)$, but it multiplies a term which is positive in the valence quark region. Thus, once again we see that the only binding effect appears in the presence of the variable x_A which is only slightly larger than x, see Eq. (2.3. This effect is too small to reproduce the data; there is essentially no EMC binding effect. The result (5.3) is very similar to Eq. (2.19) in that the term, usually associated with the binding effect, proportional to $\frac{\partial}{\partial y}F_{2N}(x_A/y)|_{y=1}$ vanishes because of the second sum rule of Eq. (1.2). We stress that any model in which nucleons are the only degrees of freedom must obey the momentum sum rule, as expressed in either Eq. (2.14) or (2.16). Thus Eq. (5.3) will emerge and the model will not have a sufficiently large binding effect to explain the nuclear deep inelastic scattering data at large values of x.

Can the conventional meson-nucleon picture of nuclear structure (which ignores off-shell effects) be used to reproduce the nuclear deep inelastic scattering data in the valence quark region of Bjorken x? The only way to get a binding effect is to compute the nuclear ground state wave function in such a manner as to obtain the mesonic $P_m^\pm$ and nucleonic contributions:

$$P^+ = P_N^+ + P_m^+, \tag{5.5}$$

$$P^- = P_N^- + P_m^-, \tag{5.6}$$

in which the meson content $P_m^\pm$ is treated explicitly. In general, $P_m^\pm$ consists of terms arising from any of the exchanged mesons which are responsible for the nuclear force:

$$P_m^\pm = P_\pi^\pm + P_\omega^\pm + P_\sigma^\pm + \cdots. \tag{5.7}$$

The equation for P^+ was used long ago [32], in which nucleons and pions contributed to the total, and with momentum conservation presented as the justification for the equation. It is useful to realize that the use of the energy-momentum tensor provides a general basis for this sum rule. For example, in the work of Refs. [8,37] the use of a chiral Lagrangian, containing isoscalar vector mesons, $V^\mu, V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$, scalar mesons ϕ and pions $\boldsymbol{\pi}$, and standard manipulations give

$$\begin{aligned} T^{++} &= V^{ik}V^{ik} + m_v^2 V^+V^+ + \bar{\psi}\gamma^+ i\partial^+\psi \\ &\quad + \partial^+\phi\partial^+\phi + \partial^+\boldsymbol{\pi}\cdot\partial^+\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\partial^+\boldsymbol{\pi}\frac{\boldsymbol{\pi}\cdot\partial^+\boldsymbol{\pi}}{\pi^2}(1 - \frac{f^2}{\pi^2}\sin^2\frac{\pi}{f}), \end{aligned} \tag{5.8}$$

with P^+ given by

$$P^+ = \langle T^{++} \rangle \Omega, \tag{5.9}$$

with the brackets denoting a ground state matrix element. The point is that each separate term corresponds to a term in either P_N^+ or P_m^+, and therefore identifiable as a term in the sum rule (5.5). However, the field equations provide relations between all of the fields. Thus, one can not get a reasonable result for P^+ by considering only one of the terms which contribute.

The pressure balance condition $P^+ = P^- = A\overline{M}$, must hold for a stable solution so that one finds

$$P_N^+ - P_N^- = P_m^- - P_m^+. \tag{5.10}$$

Thus the condition needed to prove the the Hugenholtz-van Hove theorem (that $P_N^+ = P_N^- = A\overline{M}$) is not obtained. We know that $P_N^+ < M_A$ because all of the contributions to P_m^+ are positive definite. One may therefore define a positive quantity ϵ via the deviation:

$$\epsilon \equiv \frac{P_m^+}{M_A}, \tag{5.11}$$

so that

$$\int dy\, y f_N(y) = 1 - \epsilon. \tag{5.12}$$

Thus Eqs. (5.10,5.11) can be thought of a generalization of the Hugenholtz-van Hove theorem which is equivalent to the momentum sum rule. With this new feature, the application of the expansion procedure to the integral of Eq. (2.1) yields a term proportional to ϵ:

$$F_{2A}(x) = F_{2N}(x_A) + \epsilon x_A F_{2N}'(x_A) + \gamma \left(2x_A F_{2N}'(x_A) + x_A^2 F_{2N}''(x_A) \right). \tag{5.13}$$

Equation (5.13) corresponds to the usual binding effect which now is present. However, one needs fairly large values $\epsilon \sim 0.05$ to reproduce the deep inelastic data for Iron [3], and $\epsilon \sim 0.07$ to reproduce the nuclear matter extrapolation shown in Fig. 2. Early calculations [33,32] in which pions are allowed to carry such a momentum fraction have another consequence [34]: an enhanced nuclear anti-quark content which turned out to be in contrast with the results of the nuclear Drell-Yan experiment [35]. A more recent light-front calculation [36,37], which included the effects of nucleon-nucleon correlations as well as those of an explicit meson Fock space, finds that pions carry about 2% of the nuclear plus momentum. While this value is consistent with the Drell-Yan experiment [38], using $\epsilon = 0.02$ would provide too small a reduction in the nuclear structure function. It is possible that other mesons could supply significant contributions to ϵ, and it is necessary to investigate this possibility. However, consistency with the Drell-Yan experiment must be maintained. While it seems unlikely that a careful calculation will be consistent with both the deep inelastic and Drell-Yan data, we cannot rule that out now.

The analysis of the previous paragraph is similar to the early one of Refs. [3] and [32]. The main difference occurs in the present ability [9] to compute the nuclear binding energy in terms of $P_N^\pm$ and $P_m^\pm$.

It is also worthwhile commenting on the modern calculations [6] which use the nucleon spectral function $S(p)$ to compute the quantity $f_N(y)$. It is necessary to obtain Eq. (5.12) with a significantly large value of ϵ to achieve agreement with data. However, a complete calculation should also obtain the very same value of ϵ from Eq. (5.11). But models in which the mesons are eliminated in favor of two and three nucleon potentials, forfeit the ability to compute the value of ϵ directly. It is possible to make a completely accurate calculation of $f_N(y)$ which would reproduce the correct value of ϵ, but a computational error which is only a few percent in $f_N(y)$ corresponds to a huge percentage error in the small quantity ϵ. Hence, such calculations must be regarded as inconclusive. Even if we take the calculations at face value, the models "are not completely satisfactory". If mean field models are used, nuclear binding accounts for only 20% of the observed effect [39].Very large separation energies (values of ϵ), inconsistent with the mean field calculations, are required [24,40] to reproduce the data. Calculations have been made including correlations, but the summary of Refs. [41,42] made in the book [6] is: "But for all nuclei considered the predicted deviation of the ratio $R(x)$ (is) much smaller than the experimental one." This statement applies also to the work of [43], if the "off-shell nature of the nucleon" is ignored. The inclusion of off-shell effects by allowing the nucleon structure function to depend on the momentum of the nucleon in the nucleus (as well as on x/y) can lead to a significantly improved description of the data [44,43]. This agreement is consistent with the results of the present work. Here we consistently ignore off-shell effects, categorizing these, along with a host of others, as interesting effects. In any case, one would need to understand the implications of analogous off-shell variations in operators used in impulse approximation calculations for many nuclear reactions. Furthermore, it is not clear to us that these formulations [44,43] provide nuclear structure functions which are consistent with the baryon sum rule.

VI. SUMMARY AND DISCUSSION

The principal result is that relativistic mean field models of nuclei, successful for many observables, do not contain the binding effect needed to reproduce the depletion observed by the EMC. The generality of this conclusion is related to the use of the mean field approximation, consistent with the Hugenholtz-van Hove theorem [17], which severely constrains the nucleon distribution function $f_N(y)$. This theorem has the further implication that any model in which the entire plus momentum is carried by the nucleons, in the sense of Eq. (1.2), also contains no binding effect. Thus including nucleon-nucleon correlations (within a model containing only nucleons in the Hamiltonian) cannot reproduce of the data. A minimum feature necessary to describe the data using conventional meson-nucleon dynamics is that the mesonic components must comprise an explicit part of the nuclear Fock state wave function, and the mesons must carry a significant fraction of the nuclear P^+. But there are severe constraints on the nuclear anti-quark content [34,35] and these limit the flexibility of mesonic models. Therefore, all of our present considerations are consistent with the notion that some effect not contained within the conventional framework is responsible for the EMC effect.

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APPENDIX A: FOUR MORE MODELS

Four other relativistic models are solved using light front dynamics in this section. The techniques are the same as in Section III, so our treatment will be briefer than what appears above.

1. The No Vector Walecka Model

To reduce the effects of vector mesons it is natural to employ a Lagrangian that has only one meson field, a scalar field ϕ. The repulsion is supplied by a repulsive vector point coupling. The Lagrangian is

$$\mathcal{L}_{WNV} = \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi - m_s^2\phi^2) + \overline{\psi'}(i\partial\!\!\!/ - M - g_s\phi)\psi' - Gj^\mu\, j_\mu, \tag{A1}$$

where

$$j_\mu \equiv \overline{\psi'}\gamma_\mu\psi'. \tag{A2}$$

The resulting equation of motion for the Dirac field is

$$\gamma^\mu(i\partial_\mu - 2Gj_\mu)\psi' = (M + g_s\phi)\psi', \tag{A3}$$

and the equation for the scalar field is again Eq. (3.2) The canonical energy-momentum tensor is given by

$$T^{\mu\nu} = -g^{\mu\nu}\mathcal{L} + \partial^\mu\phi\partial^\nu\phi + \frac{i}{2}\overline{\psi'}(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)\psi'. \tag{A4}$$

In the Mean Field Approximation (MFA) the equations of motion are given by Eq. (3.6) for the scalar field and

$$\gamma^\mu(i\partial_\mu - 2G\langle j_\mu\rangle)\psi' = (M + g_s\phi)\psi', \tag{A5}$$

for the Dirac field. The components of the energy-momentum tensor are given by

$$T^{++}_{MFA} = i\overline{\psi'}\gamma^+\partial^+\psi' \tag{A6}$$

$$T^{+-}_{MFA} = m_s^2\phi^2 - 2\,\overline{\psi'}(\gamma^\mu(i\partial_\mu - G\langle j_\mu\rangle) - M - g_s\phi)\psi' + \frac{i}{2}\overline{\psi'}(\gamma^+\partial^- + \gamma^-\partial^+)\psi' \tag{A7}$$

We solve the Dirac equation as in Section III using the transformation

$$\psi' = e^{-2iG\Lambda(x)}\psi, \quad \partial^+\Lambda = \langle j^+\rangle \tag{A8}$$

and define $\widetilde{j}^\mu = \langle j^\mu\rangle - \partial^\mu\Lambda$ (note that $\widetilde{j}^+ = 0$ by construction and $j^i = 0$ in the rest frame so that the only non-vanishing component in the MFA is $\widetilde{j}^- = \rho_B$). The Dirac equations for ψ_+ and ψ_- become

$$\begin{aligned}(i\partial^- - 2G\widetilde{j}^-)\psi_+ &= (\alpha_\perp \cdot p_\perp + \beta(M + g_s\phi))\psi_- \\ i\partial^+\psi_- &= (\alpha_\perp \cdot p_\perp + \beta(M + g_s\phi))\psi_+\end{aligned}$$

If we assume $\psi \sim e^{ik\cdot x}$, then we obtain

$$(i\partial^- - 2G\widetilde{j}^-)\psi_+ = \frac{k_\perp^2 + (M + g_s\phi)^2}{k^+}\psi_+. \tag{A9}$$

Returning to the energy momentum tensor, which has also changed under the transformation Eq. (A8), we find

$$T^{++}_{MFA} = 2\psi_+^\dagger(i\partial^+ + 2G\langle j^+\rangle)\psi_+ \tag{A10}$$
$$T^{+-}_{MFA} = m_s^2\phi^2 + 2\psi_+^\dagger(i\partial^- - 2G\widetilde{j}^-)\psi_+. \tag{A11}$$

Using Eq. (A9) in Eq. (A11) and the light front 4-momentum definition Eq. (3.14) we obtain

$$\frac{P^-}{\Omega} = m_s^2\phi^2 + \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+ \frac{k_\perp^2 + (M^*)^2}{k^+} \tag{A12}$$
$$\frac{P^+}{\Omega} = \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+(k^+ + 2G\rho_B) \tag{A13}$$

The second term in Eq. (A13) can be rewritten

$$\frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+ 2G\rho_B = \frac{8G\rho_B}{(2\pi)^3}\int_F d^2k_\perp dk^+ = 2G\rho_B^2$$

and the resulting equations are exactly those of the Walecka model and we draw the correspondence (with $k_F = 1.42\ \mathrm{fm}^{-1}$)

$$2GM^2 \rightarrow \frac{g_v^2}{m_v^2}M^2, \tag{A14}$$

which means that the saturation properties of this model are the same as those of the Walecka model.

An important difference between this model and the Walecka model is that the extra term is *not* due to vector mesons, but part of the nucleon contribution to P^+. All of the plus momentum is due to the nucleons and not the vector mesons. It is apparent that this model is consistent with the Hugenholtz-van Hove theorem. The values of $P^\pm$ are the same as those of the Walecka model for all values of k_F.

2. The Zimanyi-Moszkowski Model

The rescaled derivative coupling Lagrangian given by Zimanyi and Moszkowski [31] is known to have a smaller vector potential than the Walecka model. This model is defined by the Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathcal{ZM}} &= \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi - m_s^2\phi^2) - \frac{1}{4}V^{\mu\nu}V_{\mu\nu} + \frac{1}{2}m_v^2V^\mu V_\mu \\ &+\overline{\psi}\left(\gamma^\mu(i\partial_\mu - g_vV_\mu) - \frac{M}{1-\frac{g_s\phi}{M}}\right)\psi. \end{aligned} \tag{A15}$$

Using the methods described in Section III, one finds that the eigenvalue equation corresponding to Eq. (3.11) is given by

$$(i\partial^- - g_vV^-)\psi_+ = \frac{k_\perp^2 + (M^*)^2}{k^+}\psi_+ \tag{A16}$$

where

$$M^* = \frac{M}{1-\frac{g_s\phi}{M}} \tag{A17}$$

$$V^- = V_0 = \frac{g_v\rho_B}{m_v^2}. \tag{A18}$$

The field V^- is transformed according to Eq. (3.9). The relevant components of the energy-momentum tensor are

$$T^{++}_{MFA} = m_v^2V_0^2 + 2\psi_+^\dagger i\partial^+\psi_+ \tag{A19}$$

$$T^{+-}_{MFA} = m_s^2\phi^2 + 2\psi_+^\dagger(i\partial^- - g_vV^-)\psi_+ \tag{A20}$$

and so P^+ and P^- are given by

$$\frac{P^-}{\Omega} = m_s^2\phi^2 + \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+\frac{k_\perp^2 + (M^*)^2}{k^+} \tag{A21}$$

$$\frac{P^+}{\Omega} = m_v^2V_0^2 + \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+ k^+ \tag{A22}$$

which are superficially the same as the Walecka model, but differ in that now M^* is given by Eq. (A17). The parameters of the model are obtained by minimizing the total energy at $k_F = 1.42\ \mathrm{fm}^{-1}$, using the values

$$\frac{g_v^2}{m_v^2}M^2 = 43.2 \tag{A23}$$

$$\frac{g_s^2}{m_s^2}M^2 = 140.4, \tag{A24}$$

which corresponds to

$$M^* = 0.85M. \tag{A25}$$

The plus momentum may be decomposed using $P^+ = P^+_m + P^+_N$ (meson part and a nucleon part)

$$\frac{P^+_m}{A} = \frac{m_v^2 V_0^2}{\rho_B} = 73 \text{ MeV} \tag{A26}$$

$$\frac{P^+_N}{A} = \frac{1}{\rho_B}\frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+ k^+ = 850 \text{ MeV} . \tag{A27}$$

The nucleons carry about 92% of the total plus momentum in this model. Despite this, the Hugenholtz-van Hove theorem is satisfied because the expressions for $P^\pm$ in terms of V^0 and M^* are the same as those of the Walecka model. The only difference is the relation between M^* and ϕ. However, that relation does not enter in the derivation of the Hugenholtz-van Hove theorem presented in Section III.

3. A No Vector Zimanyi-Moszkowski Model

The next step is to modify the Zimanyi-Moszkowski Lagrangian by removing the vector mesons in favor of a current-current interaction as in Section A 1 This Lagrangian is

$$\mathcal{L}_{\mathcal{NVZM}} = \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi - m_s^2\phi^2) + \overline{\psi}\left(\gamma^\mu i\partial_\mu - \frac{M}{1-\frac{g_s\phi}{M}}\right)\psi$$
$$-G\overline{\psi}\gamma^\mu\psi\overline{\psi}\gamma_\mu\psi. \tag{A28}$$

The results follow exactly as those of Section A 1. Specifically, changing from a "vector" to "no vector" model does not affect the minimization of the energy density and therefore leaves the coupling constants (A23) and (A24) unchanged if we make the identification $2GM^2 \to (g_v^2/m_v^2)M^2$. The operator corresponding to Eq. (A9) is

$$(i\partial^- - 2G\widetilde{j}^-)\psi_+ = \frac{k_\perp^2 + (M^*)^2}{k^+}\psi_+ \tag{A29}$$

For this model P^- is exactly Eq. (A21) and we have plus momentum

$$\frac{P^+}{\Omega} = \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+(k^+ + 2G\rho_B) \tag{A30}$$

This model obeys the Hugenholtz-van Hove theorem because the values of $P^\pm$ are the same as those of the Zimanyi-Moszkowski model for all values of k_F.

4. Rusnak-Furnstahl Point Coupling Model

The point coupling Lagrangian of Ref. [12] is the modern version of the original Walecka which is connected to QCD through symmetry and naturalness. This model is defined in terms of the following densities: $j_\mu = \overline{\psi'}\gamma_\mu\psi'$, $\rho_s = \overline{\psi'}\psi'$ and $s_{\mu\nu} = \overline{\psi'}\sigma_{\mu\nu}\psi'$, so that

$$\mathcal{L}_{\mathcal{RF}} = \overline{\psi'}(i\partial\!\!\!/ - M)\psi' - \rho_s^2(\kappa_2 + \kappa_3\rho_s + \kappa_4\rho_s^2) - j_\mu j^\mu(\zeta_2 + \eta_1\rho_s + \eta_2\rho_s^2 + \zeta_2 j_\mu j^\mu)$$
$$-\partial_\mu\rho_s\partial^\mu\rho_s(\kappa_d + \alpha_1\rho_s) - \partial_\mu j_\nu\partial^\mu j^\nu(\zeta_d + \alpha_2\rho_s) - f_v(\partial^\mu j^\nu)s_{\mu\nu}.$$

The parameters are given in Ref. [12], and arguments for their naturalness in terms of effective field theory have also been presented [13].

The relevant components of the symmetric energy momentum tensor in the MFA are given by

$$T^{++}_{MFA} = i\overline{\psi'}\gamma^+\partial^+\psi' \tag{A31}$$
$$T^{+-}_{MFA} = -2\overline{\psi'}(i\partial\!\!\!/ - M)\psi' + 2\rho_s^2(\kappa_2 + \kappa_3\rho_s + \kappa_4\rho_s^2)$$
$$+2j_\mu j^\mu(\zeta_2 + \eta_1\rho_s + \eta_2\rho_s^2 + \zeta_4 j_\nu j^\nu) + \frac{i}{2}\overline{\psi'}(\gamma^+\partial^- + \gamma^-\partial^+)\psi' \tag{A32}$$

and the Dirac equation is

$$(i\partial\!\!\!/ - M)\psi' = [\rho_s(2\kappa_2 + 3\kappa_3\rho_s + 4\kappa_4\rho_s^2)$$
$$+\gamma_\mu j^\mu(2\zeta_2 + 2\eta_1\rho_s + 2\eta_2\rho_s^2 + 4\zeta_4 j_\nu j^\nu)$$
$$+j_\mu j^\mu(\eta_1 + 2\eta_2\rho_s)]\psi'. \tag{A33}$$

Note that the various densities are constants within the MFA, we may define

$$M^* \equiv M + \rho_s(2\kappa_2 + 3\kappa_3\rho_s + 4\kappa_4\rho_s^2) + j_\mu j^\mu(\eta_1 + 2\eta_2\rho_s) \tag{A34}$$
$$G_{RF} = \zeta_2 + \eta_1\rho_s + \eta_2\rho_s^2 + 2\zeta_4 j_\mu j^\mu, \tag{A35}$$

and follow Section III to obtain the equation

$$(i\partial^- - 2G_{RF}\tilde{j}^-)\psi_+ = \frac{k_\perp^2 + (M^*)^2}{k^+}\psi_+, \tag{A36}$$

and the momenta

$$\frac{P^-}{\Omega} = -2\kappa_2\rho_s^2 - 4\kappa_3\rho_s^3 - 6\kappa_4\rho_s^4 - 2\eta_1\rho_s\rho_B^2 - 4\eta_2\rho_s^2\rho_B^2 - 4\zeta_4\rho_B^4$$
$$+\frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+\frac{k_\perp^2 + (M^*)^2}{k^+} \tag{A37}$$
$$\frac{P^+}{\Omega} = 2G_{RF}\rho_B^2 + \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+ k^+ \tag{A38}$$

The derivation of Eq. (A37), involves adding and subtracting $M - M^*$ in order to use the Dirac operator (A36) in obtaining the last term. The numerical value of the effective mass is computed to be $M^* = 0.73M$ for $k_F = 1.31$ fm^{-1}.

The proof of the Hugenholtz-van Hove theorem is obtained by going through the steps analogous to those of Eqs. (3.19)-(3.31) for this updated model. One can see numerically in Table I that the model obeys the theorem. Additionally, the total energy, given by substituting Eqs. (A37) and (A38) into Eq. (3.19), is due entirely to nucleons, and therefore the model must obey the Hugenholtz-van Hove theorem.

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